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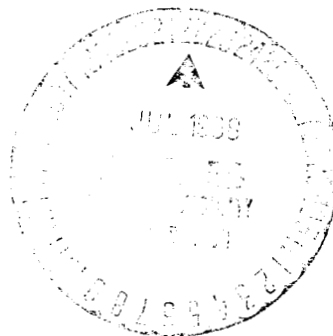
MOTION OF CHARGED PARTICLES IN AN AXIALLY ASYMMETRICAL MIRROR TRAP.

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ABSTRACT: An experimental investigation has been made of the confinement time of electrons in an axially asymmetrical magnetic mirror trap, and the value has been found of the critical nonadiabaticity parameter ρ_L/R, the numerical value of which turned out to be close to the numerical value of the nonadiabaticity parameter in the case of electron motion in an axisymmetric trap. The obtained result appears to indicate that the parameter characterizing the degree of nonadiabaticity in a trap is the ρ_L/R parameter, rather than the deviation of the magnetic field from symmetry.

The question of the motion of individual charged particles in a static spatially   /1206*
inhomogeneous magnetic field has been considered in a number of studies [1-6].

Bogolyubov and Metropol'skiy [1] used the so-called asymptotic method. If, for example, the motion of a charged particle in a magnetic trap is being dealt with, asymptotic theory asserts that with a sufficiently small value of the ρ_L/R parameter (ρ_L is the Larmor radius, R is the curvature radius of a force line of the magnetic field) the particle may make any number of oscillations between the magnetic mirrors; however, the theory does not provide for a quantitative link between the value of the small parameter ρ_L/R and the number of oscillations, but merely requires that $\rho_L/R \rightarrow 0$.

Rodionov [2] showed that: a) in the case of some configurations of the magnetic field there is a real possibility of slow, accumulating changes of the magnetic moment (10^4 -- 10^5 oscillations) which bring about emergence of the electron from the trap within a comparatively short time that depends upon the configuration of the magnetic field; b) in the case of other configurations of the magnetic field, which are easily attainable in practice, the lifetime of electrons is very great: $\sim 10^7$ oscillations. However, the physical pattern of the motion of a charged particle in a wave trap during very large time intervals remains quite unclear.

Among the first studies in which this pattern was considered were the studies of Chirikov [3, 4]. In the most general terms, the pattern of motion is represented as follows: there exists such a critical value of the ρ_L/R parameter that at greater values of ρ_L/R the motion becomes stochastic, i.e., as if random. At smaller values of ρ_L/R the motion is steady, i.e., differs little from motion with a constant magnetic moment.

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*Numbers in the margin indicate pagination in the foreign text.

Unfortunately, all the available mathematical studies have the substantial drawback that they do not provide methods of even estimating the critical value of ρ_L/R, in other words, they provide only sufficient, but in no case necessary criteria of instability.

As far as the stable domain is concerned (small values of ρ_L/R), here it is also completely unclear how long stability remains in the process of motion. Hope for rigorous clarification of this question appeared in connection with the studies of Kolmogorov [5] and Arnol'd [8], in which were found the conditions for absolute <u>/1207</u> stability, i.e., stability in the case of a finite value of a small parameter (in our case, ρ_L/R). A drawback of the theory is the absence of a precise evaluation of the critical parameter. The movement of particles in an axially asymmetrical trap is not at all subject to investigation, either by the methods of Arnol'd [8] or by any other methods known to us. Therefore we found it of interest to undertake the experimental investigation of one such system -- that of a charged particle in a magnetic trap.

During the experiments we studied the relationship between the lifetime of electrons captured in the space between the mirrors and the value of the magnetic field. Our aim was to find the numerical value of the critical parameter $(\rho_L/R)_1$, i.e., such values of ρ_L/R that when $\rho_L/R < (\rho_L/R)_1$, the lifetime of the electron is determined primarily by dispersion in the residual gas, which according to Fermi [7] is

$$\tau = \frac{\overline{\theta^2}\, W^{7/4}}{4p\ln(2{,}7\,W)} \cdot 10^{-8} \ (\text{sec}),$$ (1)

where W is the energy of the charged particles in kv, p is the pressure of the residual gas in mm Hg, while θ^2 in the case of a magnetic mirror trap is, according to [10],

$$\overline{\theta^2} = 2\left(\arccos\frac{1}{\sqrt{\gamma}} + \frac{\pi}{2} - \alpha_0\right)^2,$$

where W is the mirror ratio, and α_0 is the semiwidth of the forbidden cone.

At a value of $\rho_L/R > (\rho_L/R)_1$, according to [3-6] the lifetime of an electron is determined principally by dispersion not in residual gas, but in the inhomogeneities of the magnetic field, since now, when the electron passes along a path equal to the Larmor radius, the magnetic field changes to such a great extent that the value $\mu = W_\perp/H$ ceases to be an invariant of motion. As has been shown earlier [8], at large values of ρ_L/R the electron lifetime τ changes, as the magnetic field decreases, in accordance with the law $\tau = Ae^{BR/\rho_L}$, where $A \approx 10^{-8}$ sec, and B = 0.8 -- 0.9. In [8] there was considered the case of the motion of an electron in an axially symmetrical wave trap. As has been indicated above, the studies of Chirikov and Arnol'd (pertaining to axially symmetrical traps) make it

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possible qualitatively to trace particle motion over extended periods of time. On the other hand, the motion of particles in an axially asymmetrical trap is not subject to investigation by any method known to us.

The experiments described in [8] made it possible to calculate the critical adiabaticity parameter in the case of an axially asymmetrical trap. It was found that the value of this parameter $(\rho_L/R)_1 \cong (4.0 \pm 0.5) \times 10^{-2}$.

The aim of the present work has been to find the critical adiabaticity parameter in the case of electron motion in an axially asymmetrical magnetic trap, and to compare the value of this parameter with the value that corresponds to the case of an axially symmetrical trap. The apparatus on which the experiments were conducted consisted of a magnetic mirror trap. The maximum value of the field in the center of each of the mirrors reached 1500 oersted at a mirror ratio which varied from 2.66 to 4.44. A detailed description of the apparatus is given in [9]. Serving as the injector was an electron gun located outside the working volume behind one of the mirrors; in the center of this gun was placed a special electrode in the form of a hollow cylinder. Electron injection was effected by means of rapid variation of the electric field supplied to the electrode. The electrons captured in the space between the mirrors, which subsequently departed from the working volume due to penetration into the forbidden cone, were observed on the basis of current to the collector, which was located behind the magnetic mirror (see [9]).

In front of the collector were placed two insulated grids, to which were fed various voltages for blocking a secondary electron emission from the collector and for deriving the necessary component of the particle current from the trap. Wehn the corresponding potentials were delivered to the grids, we observed the current of the electrons which were injected by the gun, captured in the space between the mirrors, and then, having penetrated into the forbidden cone, had passed through the mirror and had gotten onto the collector. The lifetime of the electrons was considered to be not e times the time of the fall of the current pulse to the collector, but a value 3 times as great.

Axial asymmetry was created by the fact that a plate of Armco iron was placed at a distance of S from the vacuum chamber (Fig. 1).

Figure 2 shows a graph of the experimental relationship of the value of the magnetic field to the angle φ in the median plane.
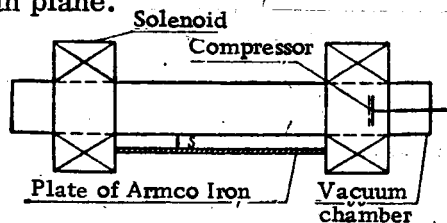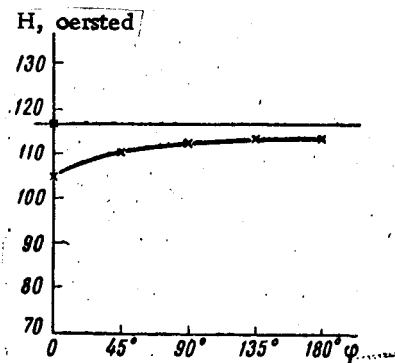

Figure 1.

Figure 2.

The presence of a gradient of the magnetic field in the direction φ leads to the fact that employment of the drift theory of electron motion in a magnetic trap

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is possible with the simultaneous satisfaction of three conditions which impose restrictions upon spatial change of the value of the magnetic field

$$\left(\frac{\rho_L}{R}\right)_{\parallel} = \frac{\rho_L}{H}\frac{V_{\parallel}}{V_{\perp}}\frac{\partial H}{\partial z} \ll 1, \tag{2a}$$

$$\left(\frac{\rho_L}{R}\right)_{\perp} = \frac{\rho_L}{H}\frac{\partial H}{\partial r} \ll 1, \tag{2b}$$

$$\left(\frac{\rho_L}{R}\right)_{\varphi} = \frac{\rho_L}{H}\frac{1}{r_0}\frac{\partial H}{\partial \varphi} \ll 1, \tag{2c}$$

where ρ_L is the Larmor radius of a particle, and $\partial H/\partial z$, $\partial H/\partial r$ and $\partial H/\partial \varphi$ are gradients of the magnetic field in the direction of the respective axis, z is the axis of symmetry of the system, r is a direction perpendicular to z, and φ is an angle in the plane of r when z = 0, r_0 is the distance of the leading center of the particle from the z axis.

When an electron moves in the trap with change of the z coordinate (the z axis coincides with the axis of symmetry of the system), the values ρ_L, R, and ρ_L/R change. Determinative in the motion of the electron is that of the values $\rho_L/R)_{\parallel}$, $(\rho_L/R)_{\perp}$, and $(\rho_L/R)_{\varphi}$, which turns out to be the greatest. Taking into account that

$$\rho_L = \frac{3.4\sqrt{W(\text{ev})}\sin\theta_0}{H(\text{oersted})}(\text{cm}),$$

we write expression (2a)$_c$ in the form:

$$\left(\frac{\rho_L}{R}\right)_{\varphi} = \frac{3.4\sqrt{W}\sin\theta_0}{H^2}\frac{1}{r_0}\frac{\partial H}{\partial \varphi} \tag{3}$$

Out of considerations of symmetry and in accordance with Fig. 2, it can be seen that the value of the Larmor radius and of the field gradient $\partial H/\partial\varphi$ reach their maximum value in the median plane (z = 0) when $\varphi \to 0$. Since the largest value of the ρ_L/R parameter is determinative in the motion of the electron, we compute the value of $(\rho_L/R)_{\varphi}$ when $z \to 0$. The maximum value of $(\rho_L/R)_{\varphi}$ we shall designate by $(\rho_L/R)_{max}$.

$$\left(\frac{\rho_L}{R}\right)_{\varphi\,max} = 3.4\sqrt{W(\text{ev})}\,H_{01}^{-2}\sin\theta_0\frac{1}{r_0}\frac{\partial H}{\partial\varphi}\bigg|_{\varphi\to 0; z\to 0}, \tag{4}$$

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where θ_0 is the angle of inclination of the electron-speed vector to the direction of the magnetic field when z = 0; r_0 is the distance of the electron from the z axis; H_{01} is the value of the magnetic field when z = 0 and $r = r_0$.

To determine the critical adiabaticity parameter $(\rho_L / R)_{\varphi_{max}}$ we find the relationship of the lifetime τ of captured electrons to the value of the magnetic field H in the center of the mirror (Fig. 3). The curves shown in Fig. 3, the experimental points of which are designated by ×, ∇, O, were plotted for electrons with an energy of W = 7.05 kv with a mirror ratio (in the absence of the plate) of Ƴ = 2.66 and a residual-gas pressure of $P = 10^{-8}$ mm Hg. First we found the curve of the relationship τ(H) in the case where the metal plate was absent (the experimental points are represented as crosses). Then we placed the metal plate at a distance of S (see Figure 1) from the chamber, and again plotted the relationship τ/(H). Then we increased the distance S between the plate and the chamber and again found the relationship τ (H). In Figure 3 are shown such curves at a distance of S = 8.7 cm and S = 7 cm. Moving the plate farther and farther from the chamber (the plate remained all the time parallel to the axis of the chamber), we found such a distance S at which the curves of relationship τ (H) with the plate (∇) and without it (×) coincided. In our case this distance turned out equal to S = 14.7 cm. Here the maximum value of the asymmetty comprised ∼1.0% (in the case of an axially symmetrical trap this value was about 0.5%). Figure 3 also shows a curve of the relationship τ (H) at the same distance S = 14.7 cm, but for $P = 3\times10^{-8}$ Hg (the experimental points are designated by a square). As can be seen from Figure 3, as the iron plate approaches the chamber, the relationships τ (H) shift into a region of stronger magnetic fields. From the graphs of Figure 3 we found that value of the magnetic field at which the lifetime of a captured electron begins to depend on the value of the magnetic field (H_1 in Figure 3), and computed the value of $(\rho_L / R)_{\varphi_{max}}$

according to expression (4). The numerical values of the adiabaticity parameter $(\rho_L / R)_{\varphi_{max}}$, computed in this manner, have been entered in the accompanying table. In the same table are entered the data of measurements carried out for two values of electron energy (W = 7.05 kv and W = 14.2 kv) and a mirror ratio (in the absence of axial asymmetry of Ƴ = 2.66.

Along the abscissa in Figure 3 was plotted the value of the magnetic field in the center of the magnetic mirror. Since in finding the realtionship τ (H) we varied the magnetic field, increasing or decreasing the current in the magnets, on the abscissa of the graphs of relationship τ (H) there was plotted the magnetic field in the center of the solenoid, since the value of the ratio of the magnetic field in the center of the coil to that at any point in the space between the mirrors is a constant.

It should be noted that at that part of curves τ (H) where they come out onto the horizontal sector, oscillograms of the current emerging from the collector are completely identical (of course, with the same initial conditions), regardless of whether the curves of relationship τ (H) are plotted for the presence of iron or for its absence. This indicates that when $\rho_L / R < (\rho_L / R)_1$,
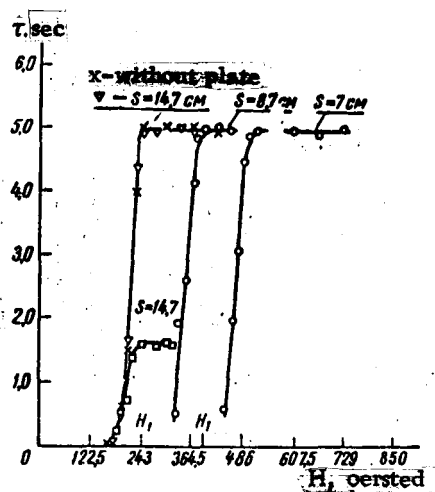
Figure 3.

the electrons are moving (from the point of view of adiabatics) in a completely identical manner.

In calculation of the value of $(\rho_L / R)_{\tau max}$ there enters into formula 4 the field in the median plane at the point where the electron is found. In the table we have cited these values of the magnetic field H_{01} at that value of the field in the center of the solenoid, at which the lifetime of a captured electron becomes a function of the magnetic field (in figure 3 this value is designated by H_1). Also cited in the table are the values of the Larmor radius at the same value of the magnetic field (H_{01}).

From the table it can be seen that as the plate approaches the chamber (i.e., when the azimuthal inhomogeneity increases), there is a decrease in the value of the Larmor radius at which the lifetime of an electron becomes a function of the magnetic field. However, in the course thereof the numerical value of the adiabaticity parameter $(\rho_L / R)_{\varphi max}$ changes almost not at all. When, however,

the azimuthal asymmetry becomes of the order of 1 - 2%, the value of $(\rho_L / R)_{\varphi max}$ becomes small (No. 4, 5, 9, 10, in the table), and the value $(\rho_L / R)_{\parallel max}$, the numerical value of which $\approx (4.0 \pm 0.5) \times 10^{-2}$, becomes determinant in the motion of an electron.

	W, вт	S, см	H_{01}	$10^4 \left(\frac{\rho_L}{R}\right)_{\varphi max}$	ρ, см	τ, сек	$10^8 \, n$
1	7.05	5.5	263	3.5	0.74	5	1
2	7.05	7.0	166	3.84	1.2	5	1
3	7.05	8.7	122	3.96	1.65	5	1
4	7.05	11.7	114	1.73	1.85	4.5	1,1
5	7.05	14.7	92	0.5	2.25	1.6	3
6	14.2	7.0	234	4.1	1.1	3.0	1.7
7	14.2	8.7	180	3.9	1.45	3.0	1.7
8	14.2	10.0	163	3.8	1.6	3.3	1.5
9	14.2	11.3	142	2	2.14	1.4	3.5
10	14.2	14.7	132	0.5	2.3	3.0	3

Thus, the data of the table indicate that with motion of an electron in an axially asymmetric mirror trap, the critical adiabaticity parameter $(\rho_L / R)_{\varphi max}$ is close to the critical adiabaticity parameter $(\rho_L / R)_{\varphi max}$ for the motion of an electron in an axially symmetical trap.

Although the adiabaticity parameter $(\rho_L / R)_{\varphi max}$ indeed does not change when

the axial asymmetry is increased, as has been shown by the experimental results cited in the table and in Figure 3, the value of the magnetic field at which the orbital magnetic moment of a particle $\mu = W_\perp / H$ ceases to be an adiabatic invariant: it increases as the axial asymmetry increases.

It should be noted that the presence of the iron plate decreases the value of the magnetic field in the region of electron capture. Increase of the magnetic field as the angle is increased takes place in the direction indicated by the arrows in figure 4. Due to the finite value of $\partial H / \partial \varphi$ the electrons will drift in an axially asymmetric field in a radial direction. This drift will not bring about the departure of electrons to the chamber walls. This can be understood from the following considerations.

Due to the presence of a radial gradient, a particle drifts azimuthally about the axis of symmetry. In the course thereof the particle gets both into the (+) region and into the (-) region. Whereas in th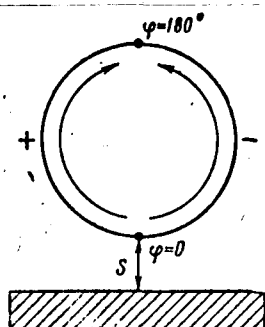e (-) region the electron drift brought about by $\partial H / \partial \varphi$ and H is directed toward the chamber wall, and the particle passes onto the magnetic surface which is located close to the wall, in the (+) region the same drift will bring about a shift of the electron to the axis of the system. Thus, in our case the presence of $\partial H / \partial \varphi$ does not create a new mechanism of the departure of electrons to the chamber walls.

Controls measurements for the determination of currents onto a cylindrical sonde located 3 mm from the chamber wall, and a comparison of oscillograms of current to the collector in the axially symmetric and the axially asymmetric traps, have shown that the cited considerations are truly valid in this case.

Figure 4.

In closing, let us note that the conducted experiments appear to permit the conclusion to be drawn that in the motion of an electron in a magnetic trap, it is not the presence or absence of axial symmetry that is essential, but only the satisfaction of conditions (2a), (2b), and (2c), and furthermore that the adiabaticity parameter ρ_L / R, in the determination of which a part is played by the gradient of the magnetic field along the field of the presence of axial asymmetry, may not be greater than $(\rho_L / R)_1 \cong (4.0 \pm 0.5) \cdot 10^{-2}$.

Thus, the parameter characterizing the degree of nonadiabaticity in a trap is apparently the ρ_L / R, parameter and not the deviation of the magnetic field from symmetry. From this it follows that for various evaluations, the ρ_L / R parameter can be used also in the case of asymmetrical traps. This conclusion can be of definite interest in connection with the fact that general theory of the motion of charged particles in axially asymmetrical traps has not yet been developed.

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